

# Supersymmetric Unification in Warped Space

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## Abstract

Supersymmetric unification in warped space provides new possibilities for model building. I argue that the picture of warped supersymmetric unification arises naturally through the AdS/CFT correspondence from the assumption that supersymmetry is dynamically broken at a scale  $\approx (10 \sim 100)$  TeV, and present several fully realistic theories in this framework. In the minimal model, the bulk  $SU(5)$  gauge group is broken at the Planck brane by boundary conditions, while supersymmetry is broken at the TeV brane. The theory preserves the successes of conventional supersymmetric unification, and yet physics at accessible energies has a drastic departure from that of the conventional scenario. There are also several variations of the minimal model giving distinct phenomenologies. These theories can provide a basis for phenomenological studies of dynamical supersymmetry breaking at low energies.

# 1 Introduction

Supersymmetry has long been the leading candidate for physics beyond the standard model. It stabilizes the Higgs potential against potentially huge radiative corrections, giving a consistent theory of electroweak symmetry breaking. The minimal construction of the supersymmetric standard model, which contains supersymmetric multiplets for the standard-model gauge and matter fields as well as two Higgs doublets, also provides an elegant picture of gauge coupling unification at a scale of  $M_X \simeq 10^{16}$  GeV. A generic prediction of supersymmetric theories – the presence of a light Higgs boson – also seems to be supported by precision electroweak data.

Despite all these successes, the construction of a supersymmetric extension of the standard model is not yet complete. The origin of supersymmetry breaking is still a mystery and the communication of supersymmetry breaking to the supersymmetric standard model (SSM) sector is not yet fully understood. Some good news, though, is that in supersymmetric theories, the non-renormalization theorem guarantees that supersymmetry can be broken only by non-perturbative effects if it is not broken at tree level. Supersymmetry, then, can be broken at a dynamical scale  $\Lambda$  of some gauge interaction  $G$  responsible for dynamical supersymmetry breaking:

$$\Lambda \sim M_{\text{Pl}} e^{-\frac{8\pi^2}{|\tilde{b}|\tilde{g}^2}}, \quad (1)$$

where  $\tilde{g}$  is the gauge coupling of  $G$  renormalized at the Planck scale, and  $\tilde{b}$  ( $< 0$ ) the beta-function coefficient for  $\tilde{g}$ . This provides a natural understanding of the smallness of the weak scale, as  $\Lambda$  is exponentially smaller than the Planck scale for  $|\tilde{b}|\tilde{g}^2 \ll 8\pi^2$  [1].

What is the scale  $\Lambda$ ? The answer depends on the mechanism by which supersymmetry breaking is mediated to the SSM sector, the sector that contains our quarks and leptons and their superpartners. If the mediation occurs through gravitational interactions,  $\Lambda \simeq 10^{10} - 10^{13}$  GeV, while if it occurs through standard-model gauge interactions,  $\Lambda$  can be much lower. These two interactions are selected as natural ways of mediation both because they are already known to exist and because they give “flavor universal” squark and slepton masses, which are needed to evade strong experimental constraints on the amount of flavor violation beyond that in the standard model [2]. Although it is possible to consider a scenario based on mediation through gravity (*e.g.* anomaly mediation), mediation by standard-model gauge interactions seems simpler to me, as it does not require a detailed understanding of Planck scale physics.

Now, suppose that the sector responsible for dynamical supersymmetry breaking (DSB) is charged under standard-model gauge interactions. Then it is possible that the gauginos obtain masses directly through their interaction with the DSB sector. The squarks and sleptons in the SSM sector then obtain flavor universal masses through standard-model gauge interactions. If this is the case, we do not need any other sector than the SSM and DSB sectors, which are in

any case needed in any supersymmetric theory. We just have to assume that the DSB sector is charged under standard-model gauge interactions, giving masses to the gauginos. What can be simpler than this?

Constructing an explicit theory along the lines described above, however, is not an easy task. Typically what happens is that if we want to make the DSB sector charged under standard-model gauge interactions,  $SU(3)_C \times SU(2)_L \times U(1)_Y$ , the gauge group  $G$  of the DSB sector becomes large, making  $SU(3)_C$  strongly asymptotically non-free, and the successful prediction for gauge coupling unification is lost [3]. In general, it is not at all easy to find an explicit gauge group and matter content for the DSB sector that does the required job and to construct a fully realistic theory. One way out from this difficulty is to further separate the DSB sector from the SSM sector by introducing fields called messenger fields, which are charged both under standard-model gauge interactions and under interactions that mediate supersymmetry breaking from the DSB sector to the messenger fields [4]. This, however, loses a certain beauty that the original picture had.

In this talk I want to present explicit theories in which the picture described above is realized in a simple way. An important new ingredient is the correspondence between 4D gauge theories and their higher-dimensional dual gravitational descriptions, especially the AdS/CFT correspondence [5]. This allows us to formulate our theories in higher dimensional spacetime, which does not require us to find the explicit gauge group and matter content for the DSB sector to construct the theories in a consistent effective field theory framework. In the construction, we require our theories to be fully realistic. In particular, we require that the successful prediction for gauge coupling unification is preserved. The theories are also free from problems of the simplest supersymmetric unified theories, such as the doublet-triplet splitting problem and the problem of overly rapid proton decay, and accommodate the successes of the conventional unification picture, such as the understanding of small neutrino masses by the see-saw mechanism. In much of the parameter space we find that the gauginos  $\lambda$  and sfermions  $\tilde{f}$  obtain masses of order  $m_\lambda \sim (\alpha/4\pi)\Lambda$  and  $m_{\tilde{f}}^2 \sim (\alpha/4\pi)^2\Lambda^2$ , respectively. This implies that the scale for supersymmetry breaking is rather low

$$\Lambda \approx 10 \sim 100 \text{ TeV}. \quad (2)$$

This may be the lowest possible scale for supersymmetry breaking we can attain in realistic supersymmetric theories.

This talk is mainly based on the works with Walter Goldberger, David Tucker-Smith, and Brock Tweedie, presented in Refs. [6, 7, 8, 9].

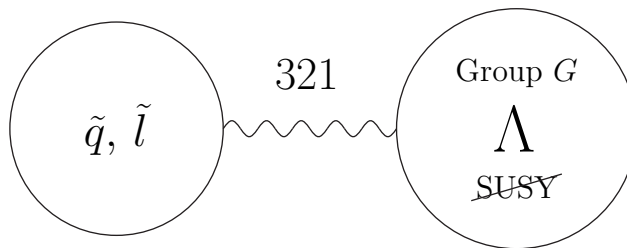


Figure 1: The basic picture of our theories.

## 2 Supersymmetry in Warped Space

The theories we consider have the following basic structure. We have a sector, the DSB sector, that breaks supersymmetry dynamically at a scale  $\Lambda \approx 10 \sim 100$  TeV. We denote the gauge group of this sector as  $G$ . This sector is also charged under the standard-model gauge group,  $SU(3)_C \times SU(2)_L \times U(1)_Y$  (321). Gaugino masses and flavor universal sfermion masses are then generated through standard-model gauge interactions. This basic picture is depicted in Fig. 1.

In general, it is not easy to find an explicit calculable theory realizing the above basic structure, since it necessarily involves a sector that is strongly coupled at the scale  $\Lambda \approx 10 \sim 100$  TeV. Suppose now that the gauge coupling  $\tilde{g}$  and the size (the number of “colors”)  $\tilde{N}$  of the group  $G$  satisfy the following relation:  $\tilde{g}^2 \tilde{N} / 16\pi^2 \gg 1$  and  $\tilde{N} \gg 1$ . In this case it is possible that the theory admits a dual higher-dimensional description that is weakly coupled, and allows explicit calculations of various quantities.

The way this duality works is the following. Let us first consider the 4D theory described in Fig. 1. In this theory the DSB sector exhibits non-trivial infrared dynamics at the scale  $\Lambda$ . Besides dynamically breaking supersymmetry, this infrared dynamics produces a series of bound states, whose typical mass scale is the dynamical scale  $\Lambda$ . Since the DSB sector is charged under 321, these bound states are also charged under 321. For  $\tilde{N} \gg 1$ , there are a large number of such bound states which are weakly coupled, as suggested by the analysis of large- $N$  QCD [10]. We now consider another theory formulated in higher dimensions, *e.g.* in 5D, and assume that the extra dimension is compactified with the characteristic mass scale for the Kaluza-Klein (KK) towers  $M_c$ . Now, suppose that the spectrum of bound states obtained in the 4D theory of Fig. 1 and the KK spectrum of this 5D theory are exactly the same,  $M_c \approx \Lambda$ , and so are any physical quantities such as the scattering amplitudes among various states. If this is the case, we can never distinguish the two theories experimentally, implying that the two theories just correspond to two different descriptions of the same physics. This is the meaning

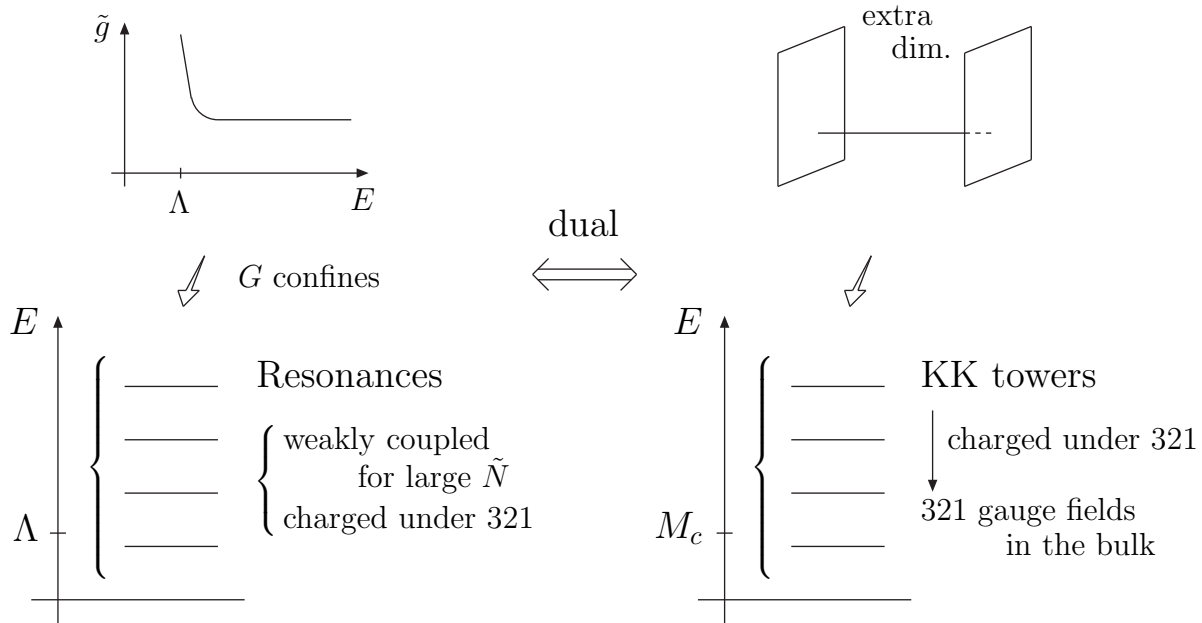


Figure 2: Schematic depiction of the correspondence between the 4D and 5D theories.

of the duality, schematically depicted in Fig. 2. Because the bound states of the 4D theory are charged under 321, the KK towers of the 5D theory should also be charged under 321. This implies that the 321 gauge fields must propagate in the 5D bulk in the “dual” 5D picture of the theory.

What the “dual” 5D theory looks like more explicitly? Let us now assume that the gauge coupling  $\tilde{g}$  evolves very slowly above  $\Lambda$ , so that the  $G$  sector is nearly conformal in a wide energy interval between  $\Lambda$  and a high scale of order  $M_X \simeq 10^{16}$  GeV. The AdS/CFT correspondence then implies that the 5D theory is formulated in anti-de Sitter (AdS) space truncated by two branes, an ultraviolet (UV) brane and an infrared (IR) brane [11]. The metric of this spacetime is then given by

$$ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (3)$$

where  $y$  is the coordinate for the extra dimension and  $k$  denotes the inverse curvature radius of the AdS space. The two branes are located at  $y = 0$  (the UV brane) and  $y = \pi R$  (the IR brane). This is the spacetime considered in Ref. [12], in which the large hierarchy between the weak and the Planck scales are generated by the AdS warp factor. We here choose the scales such that the scales on the UV and IR branes are roughly the 4D Planck scale and the scale  $\Lambda$ , respectively:  $k \sim M_5 \sim M_* \sim M_{\text{Pl}}$  and  $kR \sim 10$  (the 4D Planck scale is given by  $M_{\text{Pl}}^2 \simeq M_5^3/k$ ). Here,  $M_5$  is the 5D Planck scale, and  $M_*$  the 5D cutoff scale, which is taken to be somewhat

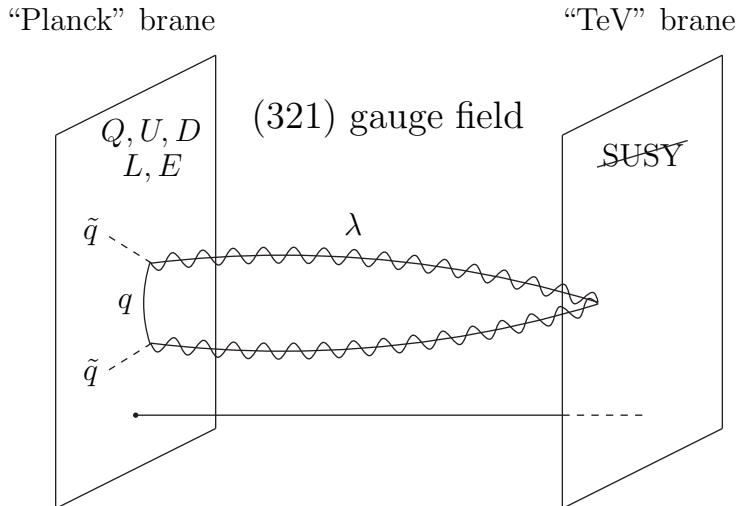


Figure 3: The overall picture for the higher dimensional description of the theory.

(typically a factor of a few) larger than  $k$ . With this choice of scales, the characteristic mass scale for the KK towers is given by  $\pi k e^{-\pi k R} \sim \Lambda \approx (10 \sim 100)$  TeV.

The theories described below [6, 7, 8, 9] are thus formulated in 5D supersymmetric warped space truncated by two branes. The structure of Fig. 1 then corresponds to breaking supersymmetry on the IR brane (also called the TeV brane) and localizing quark and lepton superfields,  $Q, U, D, L$  and  $E$ , to the UV brane (also called the Planck brane). The standard-model gauge fields propagate in the bulk. The overall picture is depicted in Fig. 3 (we can even see the similarity between the two pictures in Fig. 1 and Fig. 3). Supersymmetry breaking on the TeV brane in this picture does not have to be suppressed — it can be an  $O(1)$  breaking when measured in terms of the 5D metric of Eq. (3). Although supersymmetry breaking is directly transmitted to the 321 gauginos, the generated gaugino masses are of order TeV, because of the exponential warp factor. The squark and slepton masses are also generated through 321 gauge loops, which are flavor universal and thus do not introduce the supersymmetric flavor problem. This setup was first considered in Ref. [13]. We will see that in our theories this picture coexists with most of the successes of the conventional weak-scale supersymmetry paradigm.

Here I want to emphasize that we should not take the view that our theory has solved the hierarchy problem *twice* by introducing both supersymmetry and a warped extra dimension. Rather, the picture of a supersymmetric warped extra dimension arises if the DSB sector, which is necessarily present in any supersymmetric theory, satisfies certain conditions, *e.g.*  $\tilde{g}^2 \tilde{N}/16\pi^2 \gg 1$  and  $\tilde{N} \gg 1$ . A virtue of the higher dimensional construction is then that we do not need to know the gauge group or the matter content of the DSB sector explicitly. In fact,

once we have the picture of higher dimensional warped space and construct a theory on this space, we can forget about the “original” 4D picture for all practical purposes, although such a picture is useful for estimating various physical quantities and for obtaining insight on physical properties of the theory. Although, strictly speaking, the presence of a higher dimensional theory does not necessarily guarantee the presence of the “corresponding 4D theory”, this need not concern us. Our higher dimensional warped supersymmetric theory is a consistent effective field theory, with which we can calculate various physical quantities and compare with experiments — the theory is even weakly coupled if the cutoff scale of the theory is sufficiently larger than the AdS curvature scale.

In the next section we present a complete theory built on this picture, which accommodates the successes of the conventional weak-scale supersymmetry paradigm. Alternative, related theories will be discussed in later sections.

### 3 Warped Supersymmetric Grand Unification

We start by recalling that our DSB sector is charged under the standard-model gauge group so that it contributes to the evolution of the 321 gauge couplings. On the other hand, the successful prediction for gauge coupling unification in the minimal supersymmetric standard model (MSSM) implies that any additional contributions to the evolution of the 321 gauge couplings beyond those from the MSSM gauge, matter and Higgs fields must be universal for  $SU(3)_C$ ,  $SU(2)_L$  and  $U(1)_Y$ . In our context, this implies that the contribution from the DSB sector should be universal. This is most naturally attained if the DSB sector possesses a global  $SU(5)$  symmetry, of which the 321 gauge group of the standard model is a subgroup. In the 5D picture, this corresponds to having a gauge group (at least)  $SU(5)$  in the bulk, since the gauge symmetry in the 5D bulk corresponds to a global symmetry of the strong interacting sector in the 4D theory.

The higher dimensional unified gauge group must be broken to the 321 subgroup, as the gauge invariance in our low-energy world is 321. Because our theory looks higher dimensional, we can employ a higher-dimensional mechanism to break a gauge symmetry. In particular, we can break 5D unified gauge invariance, taken as  $SU(5)$  here, by imposing non-trivial boundary conditions on the fields at a boundary of the spacetime. This way of breaking a unified gauge symmetry has many desirable features over the conventional Higgs mechanism; for example, it allows an elegant, simultaneous solution to the problems of doublet-triplet splitting, overly rapid proton decay and unwanted fermion mass relations [14]. Here we adopt this mechanism to construct our explicit theory.

The structure of our minimal warped supersymmetric unified theory is then given as fol-

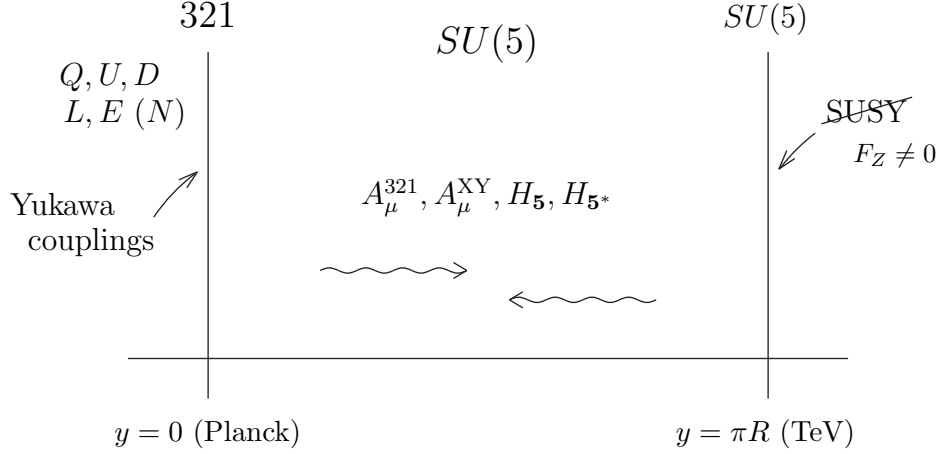


Figure 4: The overall picture of the theory.

lows [6]. The theory is formulated in 5D warped space with the metric given by Eq. (3) and the extra dimension compactified on an interval  $0 \leq y \leq \pi R$ . The bulk gauge group is taken to be  $SU(5)$ , which is broken by boundary conditions at  $y = 0$  (the Planck brane). The two Higgs hypermultiplets, which are  $\mathbf{5}$  and  $\mathbf{5}^*$  representations under  $SU(5)$ , are introduced in the bulk. Depending on the values of the bulk masses for the Higgs multiplets, which are conveniently parameterized by two dimensionless numbers  $c_H$  and  $c_{\bar{H}}$  (for notation see [6]), the wavefunctions for the zero modes arising from these multiplets can have varying shapes. The matter fields are localized to the Planck brane — they could either be located on the Planck brane or be introduced in the bulk but with the zero modes strongly localized to the Planck brane by bulk mass terms. The Yukawa couplings are also located on the Planck brane. Supersymmetry is broken on the TeV brane by a vacuum expectation value (VEV) for the auxiliary field of a chiral superfield  $Z$ . The overall picture of the theory is depicted in Fig. 4.

The boundary conditions for various fields are given more explicitly in Table 1. Here we represent the 5D gauge multiplet  $\mathcal{V} \equiv \{V, \Sigma\}$  in terms of a 4D  $N = 1$  vector superfield  $V$  and a chiral superfield  $\Sigma$ . The subscripts 321 and XY represent 321 and  $SU(5)/321$  components, respectively. A bulk hypermultiplet is represented by two 4D  $N = 1$  chiral superfields  $\Phi$  and  $\Phi^c$ . Therefore, the two Higgs hypermultiplets are denoted as  $\{H, H^c\}$  and  $\{\bar{H}, \bar{H}^c\}$ , with the subscripts  $D$  and  $T$  representing the doublet and triplet components, respectively. The boundary conditions are written in the language of orbifolding procedures. In the table we also give the boundary conditions for bulk matter fields. For bulk matter, we need two hypermultiplets  $\{T, T^c\} + \{T', T'^c\}$  in the  $\mathbf{10}$  representation and two hypermultiplets  $\{F, F^c\} + \{F', F'^c\}$  in the  $\mathbf{5}^*$  representation to complete a single generation. The subscripts for these fields denote the



$(p, p')$	gauge and Higgs fields	bulk matter fields
$(+, +)$	$V_{321}, H_D, \bar{H}_D$	$T_{U,E}, T'_Q, F_D, F'_L$
$(-, -)$	$\Sigma_{321}, H_D^c, \bar{H}_D^c$	$T_{U,E}^c, T_Q'^c, F_D^c, F_L'^c$
$(-, +)$	$V_{XY}, H_T, \bar{H}_T$	$T_Q, T'_{U,E}, F_L, F'_D$
$(+, -)$	$\Sigma_{XY}, H_T^c, \bar{H}_T^c$	$T_Q^c, T_{U,E}'^c, F_L^c, F_D'^c$

Table 1: The boundary conditions for the bulk fields. The fields written in the  $(p, p')$  column,  $\varphi$ , obey the boundary condition  $\varphi(-y) = p \varphi(y)$  and  $\varphi(-y') = p' \varphi(y')$  when we construct our space,  $0 \leq y \leq 2\pi$ , by the orbifolding procedure. Here,  $y' \equiv y - \pi R$ .

irreducible components under the 321 decompositions.<sup>1</sup>

The spectrum of the theory is obtained by KK decomposing the fields. We then find that the zero-mode sector contains only the MSSM fields: the 321 gauge field,  $V_{321}$ , two Higgs doublets,  $H_D$  and  $\bar{H}_D$ , and three generations of matter fields  $Q, U, D, L$  and  $E$ . The characteristic mass scale for the KK towers is of order TeV,  $M_c \equiv \pi k e^{-\pi k R}$ . They are almost  $N = 2$  supersymmetric and  $SU(5)$  symmetric. For example, for the gauge sector, each KK level contains  $V_{321}, \Sigma_{321}, V_{XY}$  and  $\Sigma_{XY}$ . The spectrum for a characteristic case (for characteristic values of  $c_H$  and  $c_{\bar{H}}$ ) is depicted schematically in Fig. 5. Because the spectrum at the TeV scale has a radical departure from that of the MSSM, one might wonder to what extent the successes of the conventional supersymmetric desert scenario are preserved. Below we will see that our theory preserves most of the successes of the conventional desert scenario and, moreover, is free from the problems which the minimal supersymmetric unified theory suffers from.

### 3.1 Gauge coupling unification

As is suggested from the correspondence between the 4D and 5D pictures, the evolution of the gauge couplings in our theory is logarithmic. The fact that the gauge couplings for bulk gauge fields evolve logarithmically in warped space was first noticed in Ref. [16], in which the successful prediction was also anticipated based on a heuristic argument. There have been some debates on whether theories in warped space actually allow calculations of gauge coupling unification; in particular, whether threshold corrections at an IR scale are under control (see *e.g.* [11]). Subsequent theoretical works, however, have clarified that these corrections are in fact under control, and that theories on warped space retain calculability [17]. For an observer sitting at  $y = y_*$ , physics is essentially four dimensional up to an energy  $E \sim k e^{-k y_*}$ , so for the

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<sup>1</sup>Our theory can also be viewed as one with  $SU(5)$  broken by a large VEV of a Higgs field on the Planck brane (see *e.g.* [15]), although in that case an understanding of the doublet-triplet splitting should be attributed to unknown cutoff scale physics.

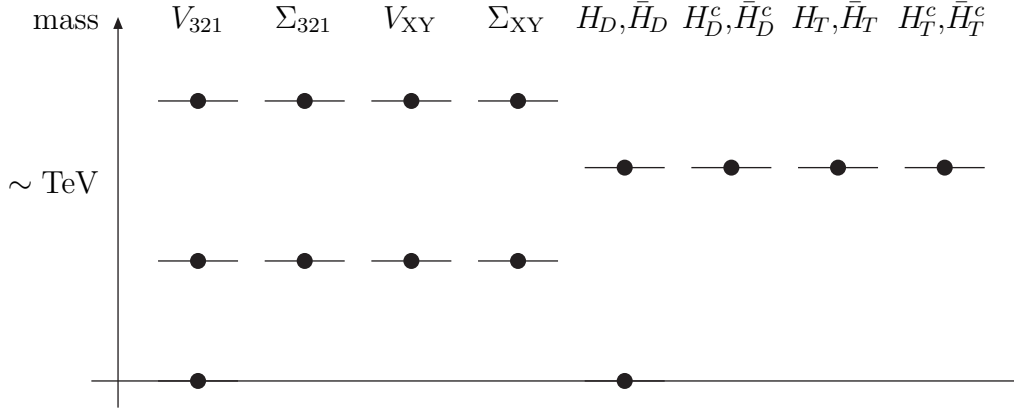


Figure 5: The schematic picture for the lowest KK spectrum.

Planck-brane observer physics is four dimensional all the way up to  $k \sim M_{\text{Pl}}$ . Now, the gauge couplings are measured, for example, by scattering two quarks — a process that occurs on the Planck brane. The evolution of the gauge couplings are then given by calculating diagrams as given in Fig 6 and summing up logarithms arising from them. At energies higher than the TeV scale  $E \gg k' \sim \text{TeV}$ , the gauge propagator in the bulk cannot probe the region close to the TeV brane, as the propagation of a gauge field from  $y = 0$  to  $y = \pi R$  receives a large suppression,  $\propto \exp(-E/k')$ , for  $E \gtrsim k'$ . In warped space all the KK modes are strongly localized to the TeV brane except for a single mode, which is often the zero mode. This implies that the contribution to the evolution of the gauge couplings at  $E \gtrsim \text{TeV}$  is dominated by the single mode and thus is logarithmic. For the case of a non-Abelian gauge field, the situation is somewhat more complicated due to the mass mixing between the different modes, but the essential physics is still the same and the evolution is still dominated by “a single mode” and is four dimensional.

In Ref. [6] we showed that the successful prediction for gauge coupling unification (the same prediction as the MSSM) is automatically obtained if the following two conditions are obeyed:

- The bulk gauge group is  $SU(5)$  (or a larger group containing  $SU(5)$ ) that is broken on the Planck brane (at the scale  $k$  or larger).
- The bulk mass parameters for the matter and Higgs fields are all larger than or equal to  $1/2$ :  $c_{\text{matter}}, c_{\text{Higgs}} \geq 1/2$ . This implies that zero modes for these fields have wavefunctions either conformally flat or localized to the Planck brane.

If the breaking scale of  $SU(5)$  is somewhat larger than  $k$ , such as the case of boundary condition breaking (breaking by the Planck-brane boundary conditions corresponds to breaking  $SU(5)$

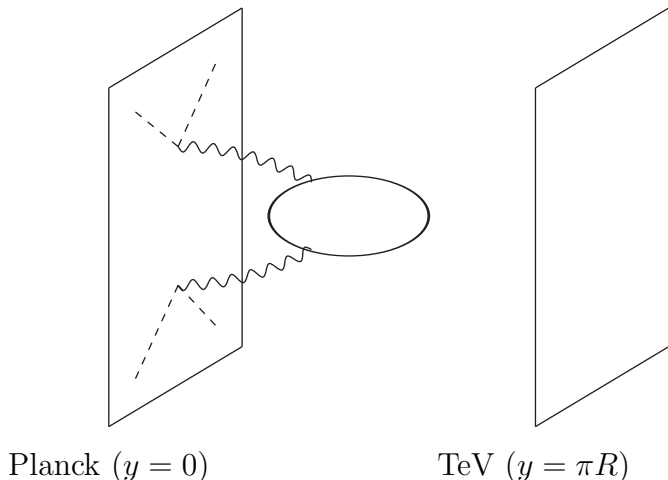


Figure 6: A diagram giving the gauge coupling evolution in warped space.

at a scale much larger than  $k$ ), tree-level operators on the Planck brane could potentially give incalculable non-universal corrections. These corrections, however, are naturally suppressed if the volume of the bulk is large, which is necessarily the case in warped space theories explaining the hierarchy between the Planck and the TeV scales. In our theory matter is localized to the Planck brane, corresponding to  $c_{\text{Matter}} \gg 1/2$ . The only remaining condition is then that the two Higgs multiplets must have mass parameters larger than or equal to  $1/2$ :  $c_H, c_{\bar{H}} \geq 1/2$ . Under this condition, we find that the prediction for the low-energy 321 gauge couplings in our theory is given by

$$\begin{pmatrix} 1/g_1^2 \\ 1/g_2^2 \\ 1/g_3^2 \end{pmatrix}_{\mu=M_Z} \simeq (SU(5) \text{ symmetric}) + \frac{1}{8\pi^2} \begin{pmatrix} 33/5 \\ 1 \\ -3 \end{pmatrix} \ln \left( \frac{k}{M_Z} \right), \quad (4)$$

which is identical to the MSSM prediction, with the AdS curvature  $k$  identified as the conventional unification scale  $M_X \simeq 10^{16}$  GeV. This determines the scales in the theory to be  $k \simeq 10^{16-17}$  GeV and  $M_* \simeq M_5 \simeq 10^{17-18}$  GeV. It is fortunate that we obtain these numbers, as we obtain roughly the correct size for the 4D Planck scale  $M_{\text{Pl}} \simeq (M_5^3/k)^{1/2}$  without introducing a new scale.

It is useful to consider gauge coupling unification in the 4D picture. In the 4D picture our theory appears as follows [6]. We have the DSB sector with the gauge group  $G$ , whose coupling  $\tilde{g}$  evolves very slowly over a wide energy interval between  $k \approx M_X \sim 10^{16}$  GeV and  $k' \approx \Lambda \sim \text{TeV}$ . The value of the coupling is  $\tilde{g} \simeq 4\pi$  in this energy interval, and the size of the gauge group  $\tilde{N}$  is sufficiently larger than 1 so that  $\tilde{g}^2 \tilde{N}/16\pi^2 \gg 1$ . The DSB sector

possesses a global  $SU(5)$  symmetry, of which the 321 subgroup is gauged and identified as the standard-model gauge group. Quark, lepton and two Higgs-doublet superfields are introduced as elementary fields, which interact with the DSB sector through 321 gauge interactions. The Higgs fields may also have direct interactions with the DSB sector through couplings of the form  $\mathcal{L} \sim H\mathcal{O}_H + \bar{H}\mathcal{O}_{\bar{H}}$ , where  $\mathcal{O}_H$  and  $\mathcal{O}_{\bar{H}}$  are operators of the DSB sector. The strengths of these couplings in the IR depend on the parameters  $c_H$  and  $c_{\bar{H}}$  in the 5D picture. Once supersymmetry is broken at the scale  $\Lambda$  by the non-trivial IR dynamics of  $G$ , the 321 gauginos, squarks and sleptons (and the Higgs fields) receive masses through 321 gauge interactions.

Since the DSB sector is charged under 321, the evolution of the 321 gauge couplings receives a contribution from this sector as well as that from the elementary states. At low energies  $q \sim \text{TeV}$ , the 321 gauge couplings are thus given by

$$\frac{1}{g_a^2(q)} = \frac{1}{g_a^2(k)} + \frac{b_{\text{DSB}}}{8\pi^2} \ln\left(\frac{k}{q}\right) + \frac{b_a}{8\pi^2} \ln\left(\frac{k}{q}\right), \quad (5)$$

where  $b_{\text{DSB}} (> 0)$  represents the contribution from the DSB sector, which is universal due to the global  $SU(5)$  symmetry, and  $b_a$  the contribution from the elementary states:  $(b_1, b_2, b_3) = (33/5, 1, -3)$ . Now, in the 4D theory dual to the 5D theory with boundary condition  $SU(5)$  breaking, the value of  $b_{\text{DSB}}$  is given such that the UV values of the 321 gauge couplings are strong,  $g_a(k) \sim 4\pi$ , in which case the first and second terms of the right-hand-side of Eq. (5) are of  $O(1/16\pi^2)$  and  $O(1)$ , respectively (the actual value is  $b_{\text{DSB}} \approx 5$ ). It is then clear that the contributions from these terms are approximately  $SU(5)$  symmetric, so that the differences of the three couplings at low energies are essentially given by the last term. This gives the same prediction for gauge coupling unification as that of the MSSM. The schematic picture for the evolution of the gauge couplings are given in Fig. 7. It is interesting to note that in our theory the hierarchy between the Planck and the weak scales are generated by

$$|\tilde{b}| \ll 1, \quad \tilde{g} \sim 4\pi, \quad (6)$$

where  $\tilde{b}$  is the beta-function coefficient for the evolution of  $\tilde{g}$ , while in the conventional picture it is generated by  $|\tilde{b}| \sim 1$  and  $\tilde{g} \ll 4\pi$  (see Eq. (1)).

### 3.2 Proton decay

Since the spectrum of the theory contains the XY gauge bosons and colored Higgs triplets at the TeV scale, one might worry that proton decay occurs at a disastrous rate in our theory. However, this should not be the case if quark and lepton fields are localized to the Planck brane. The scales on this brane do not receive a suppression by a warp factor under the dimensional reduction, so that any proton decay operator generated by integrating out baryon-number

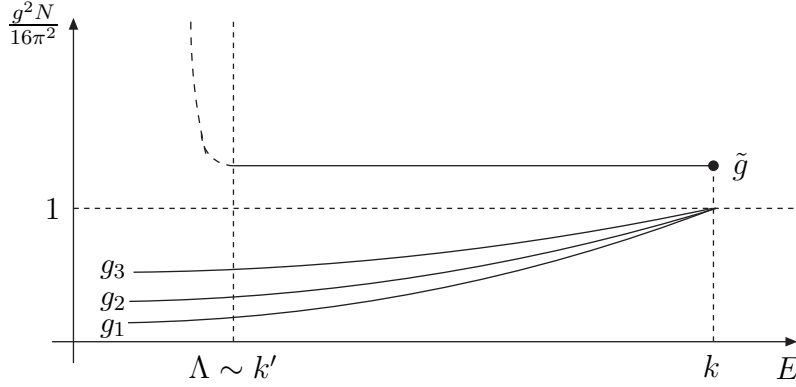


Figure 7: Schematic description for the evolution of the gauge couplings in our theory.

violating physics should be suppressed by a large mass scale of order  $k$  in the low-energy 4D theory. This must always be the case as long as the scale of  $SU(5)$  breaking is at or larger than  $k$  (the boundary condition breaking can be regarded as the breaking at the scale much higher than  $k$ ).

One might still wonder how the suppression of proton decay is explicitly realized in the KK decomposed 4D picture. To see this, note that the wavefunctions of the XY gauge bosons and the colored Higgs triplets in our theory are strongly localized to the TeV brane (this is exactly the reason why these states have TeV-scale masses — the masses for these states arise from the curvatures of the wavefunctions, which are localized to the TeV brane, so that they receive strong suppressions from a large warp factor). Therefore, the wavefunction overlaps of the XY-gauge or colored-Higgs states to the quark and lepton fields are exponentially small of  $O(\text{TeV}/M_X)$ .<sup>2</sup> This leads to tiny couplings for the baryon-number-violating vertices such as the (quark)-(lepton)-(XY gauge bosons) vertex (see Fig. 8a), and thus suppresses any proton decay process to the level of conventional unified theories, given by  $\mathcal{L}_{\text{eff}} \sim qqql/M_X^2$  or  $qq\tilde{q}\tilde{l}/M_X$ . Incidentally, the fact that the coupling in Fig. 8a is tiny does not preclude the possibility of producing the XY gauge states at colliders, as they can be produced through the coupling to the gluon, which is the QCD coupling and  $O(1)$  (see Fig. 8b). This coupling, of course, does not lead to proton decay because it conserves baryon number.

In supersymmetric theories, it is in general not sufficient to ensure that proton decay operators are suppressed by the unified mass scale  $M_X \simeq 10^{16}$  GeV, because the dimension-five operators such as  $W_{\text{eff}} \sim QQQ\tilde{L}/M_X$  could cause proton decay at a level contradicting to the

<sup>2</sup>In the theory with boundary condition  $SU(5)$  breaking, the minimal couplings of the XY gauge bosons to quarks and leptons vanish, but the couplings of the XY-gauge or colored-Higgs fields to matter can still arise in 4D from the 5D couplings that involve a derivative with respect to the fifth coordinate.

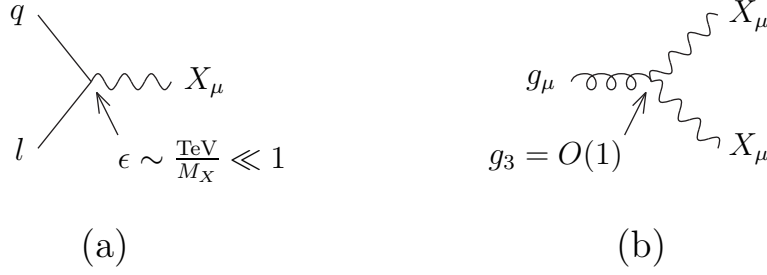


Figure 8: Vertices for the XY gauge bosons.

	$V$	$\Sigma$	$H$	$H^c$	$H$	$H^c$	$T$	$T^c$	$F$	$F^c$	$N$	$N^c$
$U(1)_R$	0	0	0	2	0	2	1	1	1	1	1	1

Table 2:  $U(1)_R$  charges for 4D vector and chiral superfields. The charges of the primed matter fields,  $T', T'^c, F', F'^c, N'$  and  $N'^c$ , are the same as the non-primed fields.

experiments. In our theory, however, dimension-five proton decay operators are simply absent because of a  $U(1)_R$  symmetry the original 5D theory possesses. The charges of various 4D  $N = 1$  superfields under  $U(1)_R$  are given in Table 2. This symmetry clearly forbids the dimension-five proton decay operators  $W \sim QQQ\bar{L}/M_X$  and  $UUDE/M_X$ , as well as other phenomenologically dangerous operators such as  $W \sim M_X H\bar{H}$ . Diagrammatically, the absence of dimension-five proton decay due to the Higgs triplet exchange is understood by the structure of the mass terms for the Higgs triplets:  $W \sim H_T H_T^c + \bar{H}_T \bar{H}_T^c$ , which is different from that in the minimal supersymmetric grand unified theory  $W \sim H_T \bar{H}_T$ . After supersymmetry is broken,  $U(1)_R$  is broken to its  $Z_{2,R}$  subgroup, which is exactly  $R$  parity. Thus, dangerous dimension-four proton decay operators are strictly forbidden, and the supersymmetric mass term for the Higgs doublets of order the weak scale can be generated. The breaking  $U(1)_R \rightarrow Z_{2,R}$  does not reintroduce dimension-five proton decay at a dangerous level.

We finally mention that unwanted fermion mass relations such as  $m_s/m_d = m_\mu/m_e$  does not arise in our theory. This is because the Yukawa couplings are located on the Planck brane, on which the gauge group is reduced to 321. An extension of the theory leading to successful  $b/\tau$  unification will be discussed in section 5.2.

### 3.3 Other issues

We have seen that in our theory two scales coexist in an intriguing way. On one hand, we have an “extra dimension” at the TeV scale, which is characterized by the appearance of

the KK towers (including those for the grand-unified states) at the TeV scale. The scale of supersymmetry breaking is also naturally set by this scale. On the other hand, we have a very high scale  $k \approx M_X \simeq 10^{16}$  GeV at which the 321 gauge couplings “unify”. This is also the scale that suppresses effective proton decay operators. The reason why the scales for gauge coupling unification and proton decay are high is that we have broken  $SU(5)$  at the Planck brane, where the warp factor is 1 and does not give any suppression of the scales. We can now go further in using these two coexisting mass scales. For example, if we introduce right-handed neutrinos  $N$  and introduce the Majorana masses and Yukawa couplings on the Planck brane, *i.e.*  $\delta(y) \int d^2\theta ((M_N/2)N^2 + y_\nu LHN)$ , we obtain small observed neutrino masses naturally through the conventional see-saw mechanism, because the Majorana masses,  $M_N$ , do not receive any suppressions from the warp factor.

In fact, the correspondence between the 4D and 5D pictures suggests that our theory can be interpreted as a purely 4D theory, in which physics between the weak and unification scales is simply 4D  $N = 1$  supersymmetric field theory. This implies, for example, that the cosmological evolution in the early universe is purely four dimensional in our theory. It is interesting to note that the theory is free from dangerous relics such as the gravitino and moduli. Because the supersymmetry-breaking scale is very low,  $\Lambda \approx (10 \sim 100)$  TeV, we expect that the gravitino (and moduli, if any) is very light

$$m_{3/2} \simeq \frac{\Lambda^2}{M_{\text{Pl}}} \approx (0.1 \sim 10) \text{ eV}, \quad (7)$$

in our theory. Such a light gravitino does not produce the “gravitino problem”, as its thermal relic abundance is small. We also note that in warped theories, the radion does not cause any cosmological problem, as its mass and interaction strengths are both dictated by the TeV scale so that it decays before the big-bang nucleosynthesis. Dark matter in our theory may come from conventional candidates such as axion, or may arise from a particle localized on the TeV brane, which has naturally TeV-scale mass and interactions and whose decay is protected by some discrete symmetry.<sup>3</sup>

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<sup>3</sup>An alternative possibility is that there is an additional source of supersymmetry breaking on the Planck brane (such a breaking may in fact help to stabilize the radius of the extra dimension). As long as the scale  $\Lambda'$  of this additional breaking is  $\Lambda' \lesssim 10^9$  GeV, this does not reintroduce the supersymmetric flavor problem, even in the presence of generic interactions between the supersymmetry-breaking field on the Planck brane and the quark-lepton supermultiplets. This allows a wide range of possibilities for the gravitino mass,  $0.1 \text{ eV} \lesssim m_{3/2} \lesssim 1 \text{ GeV}$ , some of which is consistent with the scenario that the gravitino is the super-WIMP dark matter of the universe.

## 4 Phenomenology

The phenomenology of our theory is, naturally, quite rich, as it predicts a plethora of new particles at the TeV scale — superparticles and  $SU(5)$  states as well as their KK towers (superparticles even form  $N = 2$  multiplets at higher KK level). In this section we study various aspects of the phenomenology associated with these particles.

### 4.1 Spectrum

In the minimal warped unified theory described in section 3, the spectrum of the TeV states are determined essentially by only two free parameters (up to parameters in the Higgs sector). This is because supersymmetry breaking occurs on the TeV brane, on which the gauge group is effectively  $SU(5)$ : supersymmetry breaking is transmitted to the SSM sector through the operator

$$\mathcal{L} = \delta(y - \pi R) \int d^2\theta \frac{\zeta Z}{M_*} \mathcal{W}_a^\alpha \mathcal{W}_{a,\alpha} \rightarrow \delta(y - \pi R) M_\lambda \lambda_a^\alpha \lambda_{a,\alpha}. \quad (8)$$

which has only a single coupling for  $a = SU(3)_C, SU(2)_L$  and  $U(1)_Y$  ( $M_\lambda$  does not depend on  $a$ ). This allows us to calculate soft supersymmetry breaking parameters in terms of two parameters  $x \equiv M_\lambda/k$  and  $k' = k e^{-\pi k R}$  [7]. The result of this calculation is shown in Fig. 9. In the figure, we have normalized all the masses in units of  $10 m_{\tilde{e}}$ .

In the supersymmetric limit ( $x = 0$ ), the spectrum consists of the MSSM states, which are massless, and the KK states, which are  $SU(5)$  symmetric and  $N = 2$  supersymmetric. Once supersymmetry is broken ( $x \neq 0$ ), the 321 gauginos, squarks and sleptons obtain masses. In the meantime, the masses for the KK states are also shifted; in particular, one of the two 321 gauginos at each level becomes lighter and the other heavier, and similarly for the XY gauginos. In the limit of large supersymmetry breaking ( $x \gg 1$ ), the 321 gauginos become pseudo-Dirac states by pairing up with the states that were previously the first KK excited 321 gauginos. On the other hand, the XY gaugino states become very light in this limit,  $m_{\lambda_{XY}} \propto 1/x$  — it becomes even lighter than the MSSM superparticles. These features are explained in more detail in Ref. [7].

### 4.2 4D interpretation

The characteristic features of the spectrum described above can also be understood from the 4D picture. In the 4D picture, the 321 gauginos and sfermions obtain masses, for small  $x$ , from the diagrams as shown in Fig. 10. Here, the gray discs represent contributions from the DSB sector. Using the scaling argument based on the large- $N$  expansion, the masses for the gauginos,  $M_a \equiv m_{\lambda_a^{321}}$  ( $a = 1, 2, 3$ ), are estimated as  $M_a \simeq g_a^2 (\tilde{N}/16\pi^2) \hat{\zeta} m_\rho$ , where  $g_a$  are the



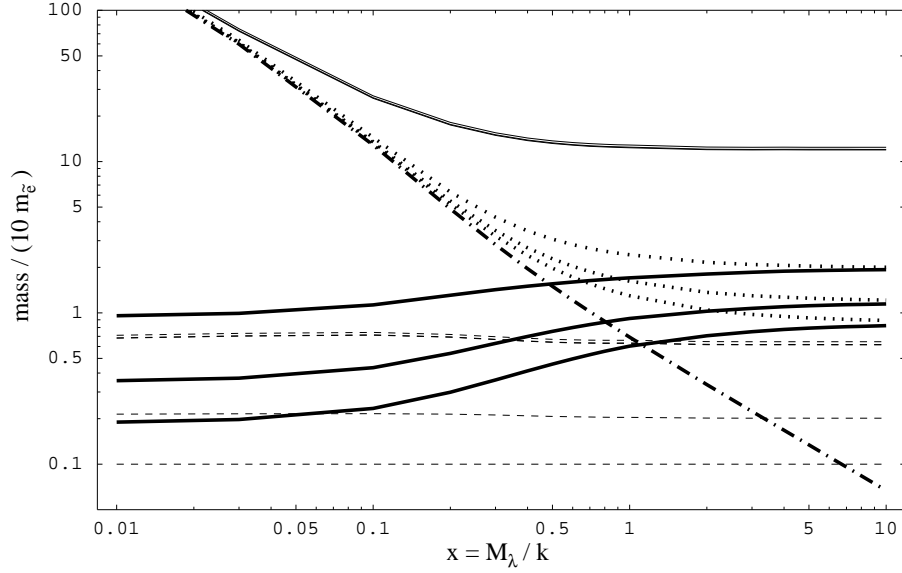


Figure 9: Masses of the MSSM scalars (dashed, with  $m_{\tilde{q}}$ ,  $m_{\tilde{u}}$ , and  $m_{\tilde{d}}$  closely spaced and  $m_{\tilde{l}}$  and  $m_{\tilde{e}}$  below), MSSM gauginos (thick solid), XY gauginos (dot-dashed), 321 gaugino KK modes (dotted), and XY and 321 KK gauge bosons (thin solid, nearly degenerate and most massive). The masses are given in units of  $10 m_{\tilde{e}}$ .

4D 321 gauge couplings,  $\hat{\zeta}$  is a dimensionless parameters of  $O(1)$ ,  $\tilde{N}$  is the size of the DSB gauge group  $G$ , and  $m_\rho$  is the typical mass scale for the resonances in the DSB sector.<sup>4</sup> Similarly, the squared masses for the scalars,  $m_{\tilde{f}}^2$ , are estimated as  $m_{\tilde{f}}^2 \simeq \sum_{a=1,2,3} (g_a^4 C_a^{\tilde{f}} / 16\pi^2) (\tilde{N} / 16\pi^2) \hat{\zeta}^2 m_\rho^2$ , where  $\tilde{f} = \tilde{q}, \tilde{u}, \tilde{d}, \tilde{l}, \tilde{e}$  represents the MSSM squarks and sleptons, and  $C_a^{\tilde{f}}$  are group theoretical factors given by  $(C_1^{\tilde{f}}, C_2^{\tilde{f}}, C_3^{\tilde{f}}) = (1/60, 3/4, 4/3), (4/15, 0, 4/3), (1/15, 0, 4/3), (3/20, 3/4, 0)$  and  $(3/5, 0, 0)$  for  $\tilde{f} = \tilde{q}, \tilde{u}, \tilde{d}, \tilde{l}$  and  $\tilde{e}$ , respectively.

To represent the gaugino and scalar masses in terms of the 5D quantities, we use the correspondence relation between the 4D and 5D theories, which are given in the present context as  $\tilde{N} / 16\pi^2 \approx 1 / g_B^2 k$  and  $m_\rho \approx \pi k'$ , where  $g_B$  represents the  $SU(5)$ -invariant 5D gauge coupling. The parameter  $\hat{\zeta}$  can be read off by matching the gaugino mass expressions of 4D and 5D theories as  $\hat{\zeta} \approx (\zeta g_B^2 F_Z / \pi M_*)$ , where the parameter  $\zeta$  appears in Eq. (8) and  $F_Z$  is the VEV of the highest component of the chiral superfield  $Z$ .<sup>5</sup> Using these relations, we obtain the following

<sup>4</sup>In a theory where  $G$  is almost conformal above the dynamical scale  $\Lambda$ , the parameter  $\tilde{N}$  may actually represent the *square* of the number of “colors” of  $G$ , and not the number of “colors” itself. Discussions on this and related issues in the AdS/CFT correspondence can be found, for example, in Ref. [18].

<sup>5</sup>The definition of  $F_Z$  here is that, in the normalization where the kinetic term of  $Z$  is canonically normalized in 4D,  $F_Z$  is defined by  $F_Z = e^{\pi k R} \partial Z / \partial \theta^2|_{\theta=\bar{\theta}=0}$ . The natural size for  $F_Z$  is then of order  $k^2 \sim M_*^2$  (no exponential suppression factor).

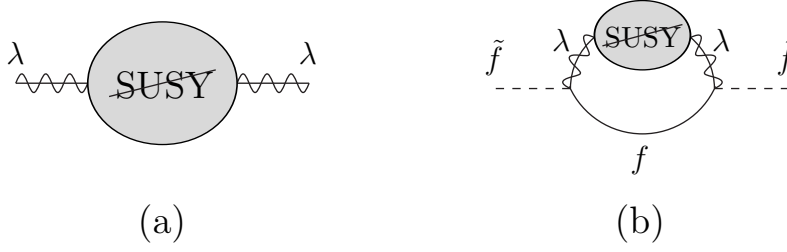


Figure 10: Examples of the diagrams that give (a) gaugino masses and (b) sfermion masses, where  $\lambda$ ,  $\tilde{f}$  and  $f$  represent gauginos, sfermions and fermions, respectively.

simple expressions for the gaugino and scalar masses:

$$M_a = g_a^2 \frac{\zeta F_Z k'}{M_* k}, \quad (9)$$

and

$$m_{\tilde{f}}^2 = \gamma \sum_{a=1,2,3} \frac{g_a^4 C_a^{\tilde{f}}}{16\pi^2} (g_B^2 k) \left( \frac{\zeta F_Z k'}{M_* k} \right)^2, \quad (10)$$

where  $g_a$  are the 4D gauge couplings given by  $1/g_a^2 = \pi R/g_B^2 + 1/\tilde{g}_{0,a}^2$  ( $1/\tilde{g}_{0,a}^2$  are the renormalized coefficients for the Planck-brane gauge kinetic terms), and  $\gamma$  is a numerical coefficient of  $O(1)$ . Note that the quantity  $(\zeta F_Z/M_*)(k'/k)$ , appearing in Eqs. (9, 10) and setting the overall mass scale, is of  $O(M_* e^{-\pi k R}/16\pi^2)$ , which is naturally of  $O(100 \text{ GeV} \sim 1 \text{ TeV})$ .

For the case of strong supersymmetry breaking, i.e.  $\zeta F_Z/k^2 \gg 1$ , the 321 gauginos become (pseudo-)Dirac states, where the extra degrees of freedom that pair up with the MSSM gauginos arise from the strong  $G$  dynamics. In this case, the diagram giving the gaugino masses is the one that mixes elementary and composite states, instead of Fig. 10a, so that the gaugino masses are given by  $M_a \simeq g_a(\sqrt{\tilde{N}}/4\pi) m_\rho \simeq (g_a/\sqrt{g_B^2 k})(\pi k')$ . The scalar masses are still given by the diagram of Fig. 10b, but now with the insertion parameter  $\hat{\zeta}$  replaced by 1, as the physics does not depend much on the strength of brane supersymmetry breaking in the limit  $\zeta F_Z/k^2 \gg 1$ . This gives  $m_{\tilde{f}}^2 \simeq \sum_{a=1,2,3} (g_a^4 C_a^{\tilde{f}}/16\pi^2)(\tilde{N}/16\pi^2) m_\rho^2 \simeq \sum_{a=1,2,3} (g_a^4 C_a^{\tilde{f}}/16\pi^2)(1/g_B^2 k)(\pi k')^2$ . These results, together with the formulae in Eqs. (9, 10), explain almost all the features observed in Refs. [6, 7, 19] for the superparticle mass spectrum in warped unified theories.

### 4.3 Grand unified particles at colliders

In our theory, grand unified particles such as the XY gauge bosons and color-triplet Higgs bosons are present at the TeV scale. What are experimental signatures of these particles? Studying the bulk Lagrangian of the theory, we find that it possesses the  $Z_2$  parity under which

all the MSSM states and their KK towers are even while the other “grand unified theory (GUT) states” are odd:

$$\begin{aligned}\text{MSSM fields (+)} : & \quad V_{321}, H_D, Q, L, \dots, \\ \text{GUT fields (-)} : & \quad V_{XY}, H_T, \dots,\end{aligned}$$

which we call the GUT parity. This parity is not broken by the couplings present in the theory such as the Yukawa couplings and supersymmetry breaking operators. It can thus be an unbroken symmetry of the theory. If this is the case (at least approximately), the lightest GUT particle (LGP) is stable at colliders, leading to characteristic experimental signatures.<sup>6</sup>

In the theory discussed in section 3, the LGP is expected to be the lightest of the XY gauginos,  $\tilde{X}$ , and leads to the following signatures [6]. Because  $\tilde{X}$  is colored, it will hadronize after production by forming a bound state with a quark (or anti-quark). There are four mesons with almost degenerate masses:

$$T^0 \equiv \tilde{X}_\uparrow \bar{d}, \quad T^- \equiv \tilde{X}_\uparrow \bar{u}, \quad T'^- \equiv \tilde{X}_\downarrow \bar{d}, \quad T^{--} \equiv \tilde{X}_\downarrow \bar{u}, \quad (11)$$

where  $\tilde{X}_\uparrow$  and  $\tilde{X}_\downarrow$  are the isospin up and down components of the XY gauginos, respectively. The mass splittings among these states are of order MeV, so that they are all sufficiently long-lived to traverse the entire detector without decaying. This yields distinctive signals; in particular, the charged states will easily be seen by highly ionizing tracks. These states can also cause intermittent highly ionizing tracks, generated through charge/isospin exchanges with the detector materials. The reach of the LHC in the masses of these states is estimated to be roughly 2 TeV. A detailed analysis for the case of the colored Higgs LGP can be found in [20].

## 5 Alternative Theories

In this section we present a variety of theories constructed along the lines presented in the previous two sections. The diversity of models presented here is an indication of how powerful the framework of warped supersymmetric grand unification is, and of the wide variety of phenomena we can obtain in this class of theories.

### 5.1 321-321 model

In the model of the previous sections, the bulk  $SU(5)$  gauge group is broken to 321 on the Planck brane while it is unbroken on the TeV brane. We could, however, consider the case where the

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<sup>6</sup>The GUT parity can in principle be broken by the presence of certain brane operators. In the present case of matter strongly localized to the Planck brane, however, the effect of the breaking is suppressed in the low-energy 4D theory so that the LGP is still effectively stable for collider purposes. It is possible, however, that the breaking leads to the lifetime of the LGP shorter than  $\sim 1$  s, which may be important for cosmology.

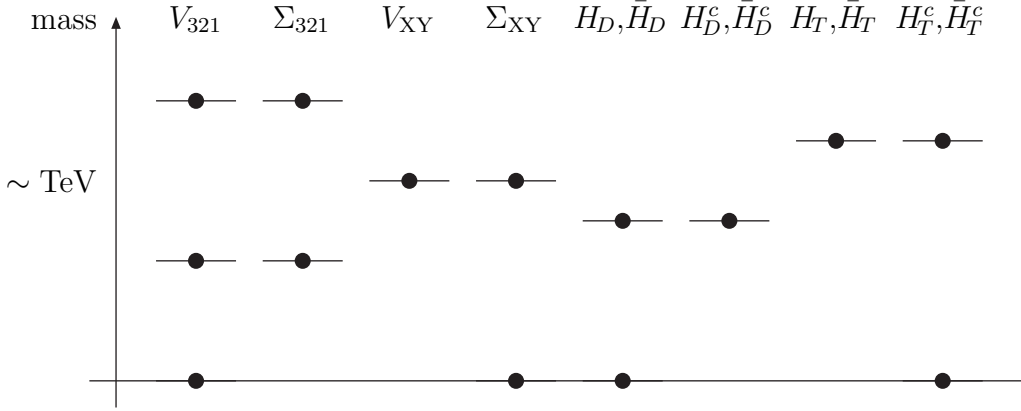


Figure 11: The schematic picture for the lowest KK spectrum of the 321-321 theories before supersymmetry breaking. After supersymmetry breaking, exotic states of  $\Sigma_{XY}$ ,  $H_T^c$  and  $\bar{H}_T^c$ , as well as the MSSM superparticles, obtain masses of order TeV.

bulk  $SU(5)$  is broken to 321 both at the Planck and TeV branes. This class of theories, called 321-321 theories, was considered in Ref. [8], where it was shown that the successful MSSM prediction for gauge coupling unification is preserved in such theories. Specifically, the boundary conditions for the bulk fields are given as  $V_{321}(+, +)$ ,  $\Sigma_{321}(-, -)$ ,  $V_{XY}(-, -)$  and  $\Sigma_{XY}(+, +)$  for the gauge sector and  $H_D, \bar{H}_D(+, +)$ ,  $H_D^c, \bar{H}_D^c(-, -)$ ,  $H_T, \bar{H}_T(-, -)$  and  $H_T^c, \bar{H}_T^c(+, +)$  for the Higgs sector (realistic theories could also be constructed with  $H_D, \bar{H}_D(+, -)$ ,  $H_D^c, \bar{H}_D^c(-, +)$ ,  $H_T, \bar{H}_T(-, +)$  and  $H_T^c, \bar{H}_T^c(+, -)$ ). The quark and lepton superfields are localized on the Planck brane, and proton decay is adequately suppressed. In the 4D picture, these theories have a similar structure to that of the theory in section 2 (e.g. that given in Fig. 1), but now the global  $SU(5)$  symmetry of the DSB sector is spontaneously broken at  $\Lambda$  by the IR dynamics of  $G$ .

As discussed in [8], this class of theories has the following distinctive features.

- Before supersymmetry breaking, the massless sector of the model contains not only the MSSM states but also exotic grand unified states  $\Sigma_{XY}$ ,  $H_T^c$  and  $\bar{H}_T^c$  (see Fig. 11). Despite the presence of these exotic states, the successful MSSM prediction for gauge coupling unification is preserved. This is because in the 5D picture the wavefunctions of the exotic states are strongly localized to the TeV brane so that they do not contribute to the running defined through the Planck-brane correlators, and in the 4D picture the exotic states are composite and so do not contribute to the running above TeV.
- After supersymmetry breaking, the exotic states, as well as the MSSM superparticles, obtain TeV-scale masses through operators localized on the TeV brane. Because the

wavefunctions of the exotic states are strongly localized to the TeV brane, while the MSSM states are not, the masses for the exotic states are an order of magnitude larger than those of the MSSM superparticles. An exception is the  $A_5^{XY}$  state, the fifth component of the XY gauge bosons (the imaginary part of the lowest component of  $\Sigma_{XY}$ ), whose mass is forbidden at tree level by higher dimensional gauge invariance. The mass of this state is generated at loop level so that it could be as light as the MSSM superparticles. Thus the LGP is  $A_5^{XY}$ , which is expected to be stable at colliders. In fact, we can understand the lightness of  $A_5^{XY}$  in the 4D picture, as it is the pseudo-Goldstone boson for the breaking  $SU(5) \rightarrow 321$  caused by the  $G$  dynamics.

- Because supersymmetry is broken at the TeV brane where the gauge group is only 321, the generated superparticle masses are non-unified. In particular, the masses for the three MSSM gauginos are completely free parameters in these theories (because the coupling  $\zeta$  in Eq. (8) can take different values for  $SU(3)_C$ ,  $SU(2)_L$  and  $U(1)_Y$ ). Squark and slepton masses are also non-unified, although they are still flavor universal. In fact, the superparticle masses are given by Eqs. (9, 10) with  $\zeta$  replaced by  $\zeta_a$  (i.e.  $\zeta$  now depends on the gauge group). In the 4D picture, we can understand the non-unified feature of the spectrum as the result of the spontaneous breakdown of the global  $SU(5)$  symmetry in the DSB sector.

## 5.2 Warped supersymmetric $SO(10)$

It is possible to extend the bulk gauge group to a larger unified group in warped supersymmetric grand unification. In particular, we can extend the bulk group from  $SU(5)$  to  $SO(10)$ . The bulk  $SO(10)$  is then broken to 321 at the Planck brane by a combination of boundary condition and Higgs breaking, while at the TeV brane it can either be unbroken or broken to the  $SU(4)_C \times SU(2)_L \times SU(2)_R$  (422) subgroup. These theories were considered in Ref. [9], and have the following (desirable) features.

- The theories provide an elegant understanding of the matter quantum numbers in terms of 422. In particular, the quantization of hypercharges can always be understood no matter how matter fields are introduced. Despite the presence of 422, realistic quark and lepton masses and mixings are easily obtained through higher dimension operators.
- Small neutrino masses are obtained quite naturally, as the seesaw mechanism arises as an automatic consequence of the theories.
- The successful prediction for  $b/\tau$  Yukawa unification can be reproduced. The ratio of the VEVs for the two Higgs doublets,  $\tan\beta \equiv H_D/\bar{H}_D$ , is naturally predicted to be large,  $\tan\beta \approx 50$ .

- In the case where the gauge group on the TeV brane is 422 and the breaking of left-right symmetry is spontaneous, we have a non-trivial relation among the three MSSM gaugino masses:  $M_1/g_1^2 = (2/5)(M_3/g_3^2) + (3/5)(M_2/g_2^2)$ . This relation still leaves room for the gaugino masses to differ from the ones expected from the conventional grand-unified mass relations for the gauginos.

### 5.3 Model with heavy Higgs boson

The construction of warped supersymmetric unification can also be used to construct supersymmetric theories in which the mass of the lightest Higgs boson is much larger than the conventional upper bound of  $\approx 130$  GeV [21]. The basic idea is to introduce two sets of Higgs doublets — one localized on the TeV brane receiving a large quartic coupling from the TeV-brane superpotential term  $W = \lambda SH\bar{H}$ , and the other propagating the bulk having the Yukawa couplings to the quarks and leptons located on the Planck brane. The Higgs doublets responsible for electroweak symmetry breaking are linear combinations of these two sets, thus having both the Yukawa couplings and a large quartic coupling. We can then obtain the mass of the lightest Higgs boson as large as  $\approx 200$  GeV, keeping the successful MSSM prediction for gauge coupling unification. This class of theories allows the possibility of a significant reduction in the fine-tuning needed for correct electroweak symmetry breaking.

## 6 Conclusions

Supersymmetric unification in warped space provides new possibilities for model building. The picture of warped supersymmetric unification arises naturally through the AdS/CFT correspondence from the assumption that supersymmetry is dynamically broken at  $\Lambda \approx (10 \sim 100)$  TeV by gauge dynamics  $G$  having certain special properties. In the minimal model [6], the bulk gauge group is  $SU(5)$  broken to the 321 subgroup at the Planck brane. The theory leaves many of the most attractive features of conventional unification intact. In particular, the successful MSSM prediction for gauge coupling unification is preserved, and small neutrino masses are naturally obtained from the seesaw mechanism. Proton decay is also naturally suppressed at a level consistent with experiments. Yet physics at accessible energies could be quite different than in the conventional scenario. The model reveals its higher-dimensional nature near the TeV scale, through the appearance of KK towers and an  $N = 2$  supermultiplet structure. The spectrum of these particles are tightly constrained, so that several definite predictions can be drawn with distinct experimental signatures.

I have also presented several variations of the minimal model [8, 9, 21], which give distinct phenomenologies. These theories differ in the gauge groups of the bulk and branes and/or in

locations of the Higgs (and matter) fields. Taking together, these theories, including the minimal one, provide a basis for phenomenological studies of dynamical supersymmetry breaking at low energies.

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